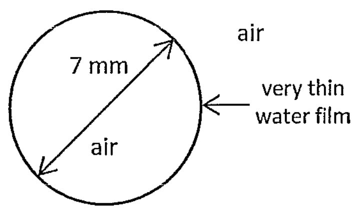
AESB2320, 2015-16
Part 1 Re-Examination - 30 June 2016

Write your solutions *on your answer sheet*, not here. In all cases ***show your work.***
Beware of unnecessary information in the problem statement.

To avoid any possible confusion,
<u>state the equation numbers and figure numbers</u> of equations and figures you use <u>along with the text you are using (BSL1, BSL2 or BSLK).</u>

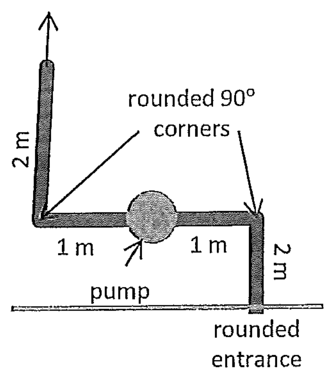
1. I recently saw a soap bubble, about 7 mm in diameter, that was slowly settling downward in air at a velocity of about 5 mm a second. The properties of air are given below. The weight of the soap film around the air in the bubble gives it an average density a little greater than that of air.
   a. Based on this information, what is the average density of the bubble? (20 pts)
   b. The average density of the bubble is the result of a spherical water film surrounding the air and the air inside. Water has density 1000 kg/m^3. Based on your answer to part (a), how thick is the water film around the bubble, to give the average density you compute in part (a)? If you did not finish part (a), show clearly how you would compute this from the average density. Don't spend too long on this part if you don't get it. Also, don't worry if the film diameter you calculate is small; soap films are very thin. (5 pts)



(25 points total)

<u>properties of air</u>
$\mu = 1.75\text{x}10^{-5}$ Pa s $\qquad \rho = 1.26$ kg/m^3

2. A fan and piping system is designed to draw air through the ceiling and out of a laboratory at a rate of 0.05 m^3/s. There is a rounded constriction at the entrance to the pipe, two rounded 90°angles in the pipe, and lengths of pipe (20 cm diameter) as shown in the figure. Pressure at both the inlet and the outlet of the pipe is at 1 atm. Assume the same properties of air as in Problem 1, and assume air is incompressible (i.e., that its density is constant). The roughness of the pipe wall is about ½ mm in scale. What rate of work must the fan do on the air to maintain this flow, in Watts?



Note that because air has such low density, you can neglect gravity in this problem.
(40 points)

3. An engineer is attempting to measure the permeability of a cylindrical rock sample by pumping water through it. The sample is 30 cm long and 5 cm in diameter, as shown below. The sample is horizontal; the water flow rate is $6x10^{-7}$ m^3/s (about 36 ml/min.) and the pressure difference is $2x10^4$ Pa.

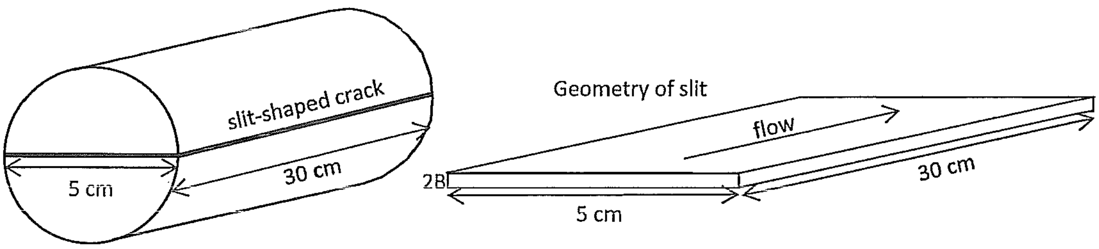
   The engineer doesn't realize it, but the rock is cracked; *all the flow is through the crack*, not through the rock itself. Assume that the crack is a smooth rectangular slit, 5 cm wide, 30 cm long, with unknown gap width 2B, as shown. The flow rate and pressure difference are as given above. The properties of water are given below.

   a. What is the gap width 2B of the fracture that would explain this flow rate at this pressure difference? Assume laminar flow.
   b. *Is* the assumption of laminar flow justified? Briefly justify your answer.

   (25 points)

<u>properties of water</u>

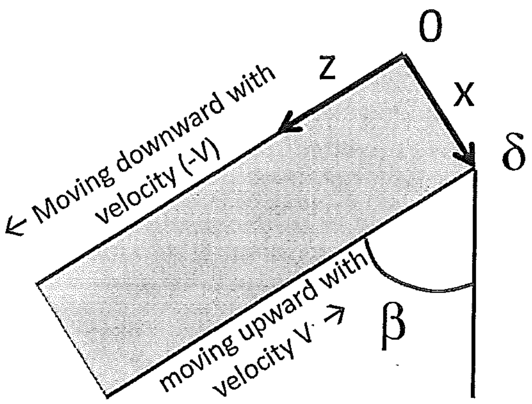
$\mu = 0.001$ Pa s $\qquad \rho = 1000$ kg/m^3



4. A Bingham plastic, with properties ρ, τ_o, μ_o, fills the gap between two parallel smooth plates held at an angle β to the vertical. The top plate is moving downward (in the negative z direction in the coordinate system shown) with velocity (-V). The bottom plate is moving upward with velocity V.

   Attached to his exam are the pages from BSL1 with the derivation for the falling-film problem. What is the *last* equation in that derivation that can be applied directly to this problem?

   (10 points)



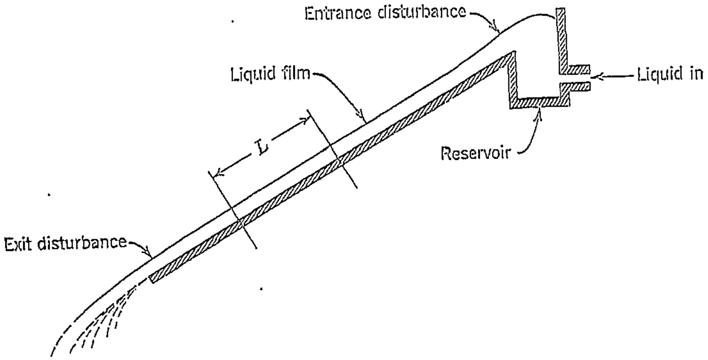


Fig. 2.2-1. Schematic diagram of falling film experiment, illustrating end effects. In the region of length L the velocity distribution is fully developed.

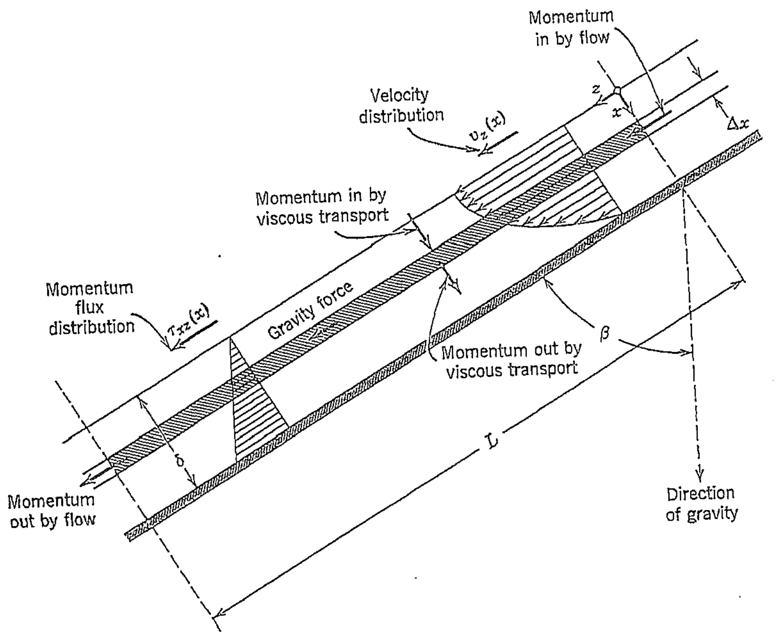


Fig. 2.2-2. Flow of a viscous isothermal liquid film under the influence of gravity with no rippling. Slice of thickness Δx over which momentum balance is made. The y-axis is pointing outward from the plane of paper.

for the velocity distribution. The integration of these two differential equations yields, respectively, the momentum flux and velocity distributions for the system. This information can then be used to calculate various other quantities, such as average velocity, maximum velocity, volume rate of flow, pressure drop, and forces on boundaries.

In the integrations mentioned above, several constants of integration appear, which are evaluated by the use of "boundary conditions," that is, statements of physical facts at specified values of the independent variable. The following are the most used boundary conditions:

a. At solid-fluid interfaces the fluid velocity equals the velocity with which the surface itself is moving; that is, the fluid is assumed to cling to any solid surfaces with which it is in contact.

b. At liquid-gas interfaces the momentum flux (hence the velocity gradient) in the liquid phase is very nearly zero and can be assumed to be zero in most calculations.

c. At liquid-liquid interfaces the momentum flux perpendicular to the interface, and the velocity, are continuous across the interface. (In the notation of §A.5, v and $np + [n \cdot \tau]$ are continuous for planar interfaces).

All three types of boundary conditions are encountered in the sections that follow.

In this section we have endeavored to present some general rules for solving elementary viscous flow problems. We now proceed to illustrate the application of these rules to a number of simple flow systems.

§2.2 FLOW OF A FALLING FILM

As our first example, we consider the flow of a fluid along an inclined flat surface, as shown in Fig. 2.2-1. Such films have been studied in connection with wetted-wall towers, evaporation and gas absorption experiments, and application of coatings to paper rolls. We consider the viscosity and density of the fluid to be constant. We focus our attention on a region of length L, sufficiently far from the ends of the wall that the entrance and exit disturbances are not included in L, that is, in this region the velocity component v_z does not depend on z.

We begin by setting up a z-momentum balance over a system of thickness Δx, bounded by the planes $z = 0$ and $z = L$, and extending a distance W in the y-direction. (See Fig. 2.2-2.) The various components of the momentum balance are then

rate of z-momentum in across surface at x	$(LW)(\tau_{xz})\|_x$	(2.2-1)

rate of z-momentum out across surface at $x + \Delta x$	$(LW)(\tau_{xz})\|_{x+\Delta x}$	(2.2–2)
rate of z-momentum in across surface at $z = 0$	$(W\Delta x\, v_z)(\rho v_z)\|_{z=0}$	(2.2–3)
rate of z-momentum out across surface at $z = L$	$(W\Delta x\, v_z)(\rho v_z)\|_{z=L}$	(2.2–4)
gravity force acting on fluid	$(LW\Delta x)(\rho g \cos \beta)$	(2.2–5)

Note that we always take the "in" and "out" directions in the direction of the positive x- and z-axes (in this problem these happen to coincide with the direction of momentum transport). The notation $|_{x+\Delta x}$ means "evaluated at $x + \Delta x$."

When these terms are substituted into the momentum balance of Eq. 2.1–1, we get

$$LW\tau_{xz}|_x - LW\tau_{xz}|_{x+\Delta x} + W\Delta x\, \rho v_z^2|_{z=0} - W\Delta x\, \rho v_z^2|_{z=L} + LW\Delta x\, \rho g \cos \beta = 0 \quad (2.2\text{–}6)$$

Because v_z is the same at $z = 0$ as it is at $z = L$ for each value of x, the third and fourth terms just cancel one another. We now divide Eq. 2.2–6 by $LW\,\Delta x$ and take the limit as Δx approaches zero:

$$\lim_{\Delta x \to 0} \left(\frac{\tau_{xz}|_{x+\Delta x} - \tau_{xz}|_x}{\Delta x} \right) = \rho g \cos \beta \quad (2.2\text{–}7)$$

The quantity on the left side may be recognized as the definition of the first derivative of τ_{xz} with respect to x. Therefore, Eq. 2.2–7 may be rewritten as

$$\frac{d}{dx} \tau_{xz} = \rho g \cos \beta \quad (2.2\text{–}8)$$

This is the differential equation for the momentum flux τ_{xz}. It may be integrated to give

$$\tau_{xz} = \rho g x \cos \beta + C_1 \quad (2.2\text{–}9)$$

The constant of integration may be evaluated by making use of the boundary condition at the liquid-gas interface (see §2.1):

B.C. 1: at $x = 0$, $\tau_{xz} = 0$ (2.2–10)

Substitution of this boundary condition into Eq. 2.2–9 reveals that $C_1 = 0$. Hence the momentum-flux distribution is

$$\boxed{\tau_{xz} = \rho g x \cos \beta} \quad (2.2\text{–}11)$$

as shown in Fig. 2.2–2.

If the fluid is Newtonian, then we know that the momentum flux is related to the velocity gradient according to

$$\tau_{xz} = -\mu \frac{dv_z}{dx} \quad (2.2\text{–}12)$$

Substitution of this expression for τ_{xz} into Eq. 2.2–11 gives the following differential equation for the velocity distribution:

$$\frac{dv_z}{dx} = -\left(\frac{\rho g \cos \beta}{\mu}\right) x \quad (2.2\text{–}13)$$

This equation is easily integrated to give

$$v_z = -\left(\frac{\rho g \cos \beta}{2\mu}\right) x^2 + C_2 \quad (2.2\text{–}14)$$

The constant of integration is evaluated by using the boundary condition

B.C. 2: at $x = \delta$, $v_z = 0$ (2.2–15)

Substitution of this boundary condition into Eq. 2.2–14 shows that $C_2 = (\rho g \cos \beta / 2\mu)\delta^2$. Therefore, the velocity distribution is

$$\boxed{v_z = \frac{\rho g \delta^2 \cos \beta}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]} \quad (2.2\text{–}16)$$

Hence the velocity profile is parabolic. (See Fig. 2.2–2.)

Once the velocity profile has been found, a number of quantities may be calculated:

(i) The *maximum velocity* $v_{z,\max}$ is clearly the velocity at $x = 0$; that is

$$v_{z,\max} = \frac{\rho g \delta^2 \cos \beta}{2\mu} \quad (2.2\text{–}17)$$